A Coupling of Pseudo Nambu Goldstone Bosons to Other Scalars and Role in Double Field Inflation

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Abstract

We find a coupling of Pseudo Nambu Goldstone bosons (PNGBs) to other (ordinary) scalars, and consider its importance in various contexts. Our original motivation was the model of Double Field inflation [1]. We also briefly consider the role of this coupling for the case of the QCD axion.

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The inflationary scenario was proposed in 1981 by Guth [2] to explain several unresolved problems of the standard hot Big Bang cosmology, namely the horizon, flatness and monopole problems. During the inflationary epoch, the energy density of the universe is dominated by vacuum energy, $\rho \sim \rho_{vac}$, and the scale factor of the universe expands superluminally, $\ddot{R} > 0$. In many models the scale factor grows as $R(t) \propto e^{Ht}$, where the Hubble parameter $H = \dot{R}/R \sim (8\pi G\rho_{vac}/3)^{1/2}$ during inflation. If the interval of accelerated expansion satisfies $\Delta t \geq 60H^{-1}$, a small causally connected region of the universe grows sufficiently to explain the observed homogeneity and isotropy of the universe, to dilute any overdensity of magnetic monopoles, and to flatten the spatial hypersurfaces, $\Omega \equiv 8\pi G\rho/3H^2 \rightarrow 1$.

In the original model of inflation [2], now referred to as Old Inflation, the universe supercools to a temperature $T \ll T_c$ during a first-order phase transition with critical temperature T_c. The nucleation rate for bubbles of true vacuum must be slow enough that the Universe remains in the metastable false vacuum long enough for the required ~ 60 e-foldings of the scale factor. Unfortunately, the old inflationary scenario has been shown to fail [3] because the interiors of expanding spherical bubbles of true vacuum cannot thermalize properly and produce a homogeneous radiation dominated universe after the inflationary epoch. Subsequently New Inflation [4] and many other models with rolling fields were proposed, in which the idea of a strongly first order supercooling phase transition was abandoned; these new models were able to successfully reheat back to a radiation dominated epoch after the inflation. Extended Inflation[5] and Double Field Inflation[1] were two different models that were proposed in an attempt to revive inflation via a first order phase transition.

In the Double Field model, a time-dependent nucleation rate is obtained via the coupling of two scalar fields. As shown in [1], the rolling field that catalyzes a time dependent nucleation rate for the inflaton must have a flat potential in order to avoid overproducing density fluctuations. In this paper we investigate using a PNGB for the rolling field to produce this flat potential (see also [12]).

We have several motivations for considering the Double Field model with PNGB field as the rolling field. This model will produce several features that are different from what one would obtain from a model with only one scalar field that is a rolling field as the inflaton (e.g. a PNGB as inflaton as in Natural Inflation [6]). First, the structure produced in such a model may be different, as there will be bubbles produced from the first order phase transition as well as the density fluctuations from the rolling field. It may be that the spectrum of perturbations from the rolling field will be different here. In addition, at the end of the first order transition, the possibility exists of producing topological objects such as cosmic strings. Having inflation take place via a first order phase transition is simply a different approach from the purely rolling models.

Outline: In Section I we find an interaction of a PNGB with other scalars. We postulate a coupling of the Peccei Quinn field to another scalar; at tree level there can be no coupling of the Goldstone mode (the PNGB) to scalars without violating the underlying symmetry. However, from the effective action to one loop we find that the (spatial or temporal) derivative of the goldstone field couples to the other scalar. In Section II we investigate the role of this coupling in Double Field Inflation if the rolling

field is a PNGB. In Section III we briefly mention the role this coupling would have for the case where the PNGB is a QCD axion.

The Term: Taking our cue from the axion, we describe a simple model which has all the ingredients we are interested in. We consider two scalar fields: a complex field Φ_1 and a real field Φ_2, as well as a fermion ψ. The Lagrangian density is

$$L = |\partial_\mu \Phi_1|^2 + \frac{1}{2}(\partial_\mu \Phi_2)^2 - V_{tot}(\Phi_1, \Phi_2) + i\bar{\psi}\gamma^\mu \partial_\mu \psi - (h\bar{\psi}_L \psi_R \Phi_1 + h.c.) \quad (1)$$

where $\psi_{(R,L)}$ are, respectively, right- and left- handed projections of the fermion field $\psi_{(R,L)} = (1 \pm \gamma_5)\psi/2$. We take the potential to be

$$V_{tot} = V_1(\Phi_1) + V_2(\Phi_2) + V_I(\Phi_1, \Phi_2) \ . \quad (2)$$

The action is invariant under the global chiral U(1) symmetry:

$$\psi_L \to e^{i\alpha/2}\psi_L, \psi_R \to e^{-i\alpha/2}\psi_R, \Phi_1 \to e^{i\alpha}\Phi_1 \ , \quad (3)$$

analogous to the Peccei-Quinn symmetry in axion models. Note that Φ_2 does not transform under the U(1) symmetry.

We take the symmetry to be spontaneously broken at the energy scale f, via a potential

$$V_1(\Phi_1) = \lambda_1 \Big[\Phi_1^* \Phi_1 - f^2/2\Big]^2 \ , \quad (4)$$

where the scalar self-coupling λ_1 can be of order unity. Below this scale, the scalar part of the Lagrangian is then

$$L_{scalar} = |\partial_\mu \Phi_1|^2 + \frac{1}{2}(\partial_\mu \Phi_2)^2 - m_1^2 |\Phi_1|^2 + \frac{1}{2} m_2^2 \Phi_2^2 + V_R(\Phi_1, \Phi_2) \ , \quad (5)$$

with $-m_1^2 < 0$ since the symmetry is spontaneously broken. V_R is defined to be all contributions to the potential other than the mass terms. We can write

$$\Phi_1 = (\rho + f)e^{i\chi/f} \ . \quad (6)$$

Below the scale f, we can neglect the superheavy radial mode ρ since it is so massive that it is frozen out, with mass $m_1 = (2\lambda)^{1/2} f$. The remaining light degree of freedom is the angular variable χ, the Goldstone boson of the spontaneously broken U(1) symmetry [one can think of this as the angle around the bottom of the Mexican hat described by Eq. (4)].

Subsequently the symmetry is explicitly broken at at scale Λ. For example $U(1)_{PQ}$ is explicitly broken at the QCD scale ~ 1 GeV, when instantons become the dominant contribution to the path integral. Then, for the case of QCD, the Goldstone mode χ becomes the axion. In general the potential for the PNGB below the scale Λ becomes

$$V(\chi) = \Lambda^4 [1 \pm \cos(N\chi/f)] \ , \quad (7)$$

where N is the number of minima of the potential.

Because of the $U(1)$ symmetry, the interaction potential $V_I(\Phi_1, \Phi_2)$ must be independent of χ. The potential must always be invariant under $\chi \to \chi + c$, so that the only coupling we can hope to get between the Goldstone mode χ and the field Φ_2 is through derivative couplings involving $\partial_\mu \chi$. At tree level, the kinetic term in the Lagrangian for the field Φ_1 is $L_{tree} \sim |\partial_\mu \Phi_1|^2$; note that this contains a term $\sim (\partial_\mu \chi)^2$. We will obtain one-loop corrections to this kinetic term, $L_{1-loop} \sim \left[Z(\Phi_1, \Phi_2) + 1\right] |\partial_\mu \Phi_1|^2$. Part of this one-loop correction is the interaction term we are looking for.

To obtain this we must calculate the effective *action* for the field ϕ_1 as a function of a temporally and or spatially varying classical goldstone background field. We follow Ref. [7], who found an effective action expansion in perturbation theory. One performs a derivative expansion of the functional determinant for the gaussian fluctuations around the classical background. Ref. [7] found the leading order correction to be

$$Z(\Phi_1) = 1 + \frac{1}{12(4\pi)^2} U^{-1} \left|\frac{\partial U}{\partial \Phi_1}\right|^2 , \tag{8}$$

where $U = -m_1^2 + \frac{\partial^2 V_R}{\partial \Phi_1 \partial \Phi_1^*}$ Again, V_R is defined to be all contributions to the potential other than the mass terms. Chan [7] also found higher order terms in the expansion in $\frac{\partial_\mu \chi}{f^2}$; we will assume that this ratio is small so that we can ignore higher order derivatives and consider only the leading term above.

For example, we consider $V_R = \frac{1}{4}\lambda_1 |\Phi_1|^4 + \frac{1}{24}\tilde{\lambda}_2 \Phi_2^4 + \lambda_3 |\Phi_1|^2 \Phi_2^2$. Then $U = -m_1^2 + \lambda_1 \Phi_1^2 + \lambda_3 \Phi_2^2$ and

$$Z(\Phi_1) = 1 + \frac{1}{12(4\pi)^2} [-m_1^2 + \lambda_1 |\Phi_1|^2 + \lambda_3 \Phi_2^2]^{-1} \lambda_1^2 |\Phi_1|^2 . \tag{9}$$

Thus, to one loop, our interaction term is

$$V_I(\chi, \Phi_2) = \frac{(\partial_\mu \chi)^2 \lambda_1^2 |\Phi_1|^2}{[-m_1^2 + \lambda_1 |\Phi_1|^2 + \lambda_3 \Phi_2^2] 12(16\pi^2)} . \tag{10}$$

Role of the Coupling in Double Field Inflation: In Double Field Inflation we take the field Φ_2 to be the inflaton, a field that tunnels from false to true vacuum via nucleation of bubbles at a first order phase transition. We take the potential to be an asymmetric potential with metastable minimum Φ_- and absolute minimum Φ_+ (see Fig. 1). The energy difference between the vacua is ϵ. In the zero-temperature limit, the nucleation rate Γ_N (per unit time per unit volume) for producing bubbles of true vacuum in the sea of false vacuum through quantum tunneling has the form [8]

$$\Gamma_N(t) = A e^{-S_E} , \tag{11}$$

where S_E is the Euclidean action and A is a determinantal factor which is generally of order T_C^4 (where T_c is the energy scale of the phase transition).

The basic idea of Double Field Inflation is to have a time dependent nucleation rate of true vacuum bubbles: initially the rate is virtually zero, so that the universe remains in the false vacuum for a long time and sufficient inflation takes place. Then, after this has taken place, the nucleation rate sharply becomes very large, so that bubbles of true vacuum nucleate everywhere at once and the phase transition is able to complete (unlike in Old Inflation). The universe then has a chance to thermalize and return to radiation domination. In Double Field Inflation, this sudden change in the nucleation rate is achieved by the coupling to a second scalar field, which is a rolling field. The purpose of this rolling field is to serve as a catalyst for the inflaton to go through the phase transition. While the rolling field is near the top of its potential, the nucleation rate is very small; once the rolling field nears the bottom, the nucleation rate becomes very large. Examples were given in [1], hereafter Paper I. However, density fluctuations are produced by quantum fluctuations in the rolling field; the amplitude of fluctuations is too large (in excess of microwave background measurements) unless the rolling potential is very flat, namely the ratio of height to width must be less than $O(10^{-10})$ [9]. For this reason we have investigated using a PNGB as the rolling field. A PNGB can naturally provide the required flat potential. Unfortunately, as we show below, this requires the other potential, the potential for the inflaton, to be flat as well.

Thus, using the notation above, we take Φ_2 to be the inflaton, a tunneling field, and the PNGB χ to be the rolling field. Then the overall Lagrangian is given by eqn (1) where the interaction term is given by eqn. (10). For definiteness, we take the potential of the inflaton field to be

$$V_2(\Phi_2) = \frac{1}{8}\lambda_2(\Phi_2^2 - a^2)^2 - \frac{\epsilon}{2a}(\Phi_2 - a) \ . \tag{12}$$

To leading order, the metastable minimum is given by $\phi_- = -a$ and the absolute minimum by $\phi_+ = +a$.

For the interaction term we use eqn. (10) above. We take the spatial derivatives to be zero and focus on the time derivative $\dot{\chi}$. We can take the amplitude of the rolling field to be $|\Phi_1|^2 = f^2$. Then the term becomes

$$V_I(\chi, \Phi_2) = \frac{\dot{\chi}^2 \lambda_1^2 f^2}{12(16\pi^2)[-m_1^2 + \lambda_1 f^2 + \lambda_3 \Phi_2^2]} \ . \tag{13}$$

Here we assume that the field $\Phi_2 \leq |\Phi_1| \sim f$ and expand the denominator (later we will see that sufficient inflation requires $f \sim m_{pl}$ so that this assumption is not unreasonable). Then the coupling part of the term is

$$V_I(\chi, \Phi_2) = \frac{-1}{12(16\pi^2)} \dot{\chi}^2 \lambda_1^2 f^2 \lambda_3 \Phi_2^2 \frac{1}{(-m_1^2 + \lambda_1 f^2)^2} \ . \tag{14}$$

As the rolling field χ rolls down its potential, the time derivative $\dot{\chi}$ increases, this coupling term gets more negative, the entropy S decreases, and the tunneling rate increases. Thus the tunneling rate indeed increases with time, as it must.

The equation of motion for the rolling field is $\ddot{\chi} + 3H\dot{\chi} = -\frac{dV}{d\chi}$; we will assume the slow roll limit in which the $\ddot{\chi}$ term is negligible. Then, using $V(\chi)$ from eqn. (7) (and assuming the coupling term will not change our result by many orders of magnitude), from the equation of motion we find

$$\dot{\chi} \sim \Lambda^4 \chi/(3Hf^2) \ . \tag{15}$$

Here we have made the small angle approximation $\sin(\chi/f) \sim \chi/f$, which is always true to within an order of magnitude. We take $H^2 = 8\pi M^4/3m_{pl}^2$ where M is the energy scale at which inflation takes place and $m_{pl} = 10^{19}$ GeV is the Planck mass. Thus the term is

$$V_I(\chi, \Phi_2) = -\eta\chi^2\Phi_2^2 \tag{16a}$$

where

$$\eta \simeq 10^{-5}\lambda_3 \frac{\Lambda^8 m_{pl}^2}{M^4 f^6} \ . \tag{16b}$$

In Paper I we discussed several constraints on the Double Field model. We apply these constraints here to the case where the rolling field is a PNGB with interaction term above, to find what the factor η must be.

First, we want the Φ_2 field to dominate the dynamics of the Universe and be responsible for the inflationary epoch; hence we require $V_2(\Phi_2) > V_2(\chi)$, i.e. $\epsilon \geq \Lambda^4$.

Second, in order for the coupling of the χ field to influence the inflaton and bring an end to inflation, we need the coupling term to be sufficiently large at the end of inflation, i.e. $\eta\chi^2\Phi_2^2 \sim \epsilon$. Since $\Phi_2 \sim -a$ during inflation, this requirement becomes $\eta f^2 a^2 \sim \epsilon$.

Third, the slowly rolling field must be able to roll despite the frictional effect provided by the interaction term. We must have $\dot{\chi} > 0$, i.e. $\Lambda^4/f^2 - 2\eta a^2 \geq 0$.

The combination of these three constraints means that all the terms in the potential must be roughly comparable,

$$\epsilon \sim \eta f^2 a^2 \sim \Lambda^4 \ . \tag{17}$$

Bubbles will nucleate at a rate given by Eq. (11). For the potential of eqn. (13) and in the limit that the nondegeneracy of the vacua is small (i.e. ϵ small, thin wall limit), the Euclidean action can be obtained analytically [8] and is given by

$$S_E = \frac{\pi^2}{6}\frac{\lambda_2^2 a^{12}}{\epsilon_{eff}^3} \ . \tag{18}$$

Here $\epsilon_{eff} = \epsilon + \eta\chi^2\Phi_2^2 \sim \epsilon$ from above. To obtain an appropriate nucleation rate, we need $S_E \sim 10$. Using eqns. (17) and (18) and taking $M \sim \Lambda$, we find that this requires

$$\lambda_2 \sim 10^{-15}(\Lambda/f)^6(m_{pl}/f)^6 \ , \tag{19}$$

where we have taken $\lambda_3 = 1$ and $M \sim \Lambda$. We know [9] that $\Lambda/f \leq 10^{-3}$. We then have $\lambda_2 \leq 10^{-33}(m_{pl}/f)^6$. As we have no natural reason to expect otherwise, we would like to make the coupling $\lambda_2 = 1$ by taking $f = 3 \times 10^{-6} m_{pl}$ and $\Lambda \sim 3 \times 10^{10}$ GeV.

However, it turns out that scales this low will not allow for enough inflation in a reasonable fraction of the universe [10]. Here we have the number of e-foldings $N_T = 3H^2 \int_{\chi_0}^{f} \frac{d\chi}{F}$, where the driving term $F = dV/d\chi = \Lambda^4 \frac{1}{f}\frac{\chi}{f} - \eta 2\chi a^2 \sim \chi\beta\Lambda^4/f^2$, where $\beta = O(1)$. We find $N_T \sim \frac{8\pi}{\beta}\frac{f^2}{m_{pl}^2}\ln(f/\chi_0)$. Requiring 60 e-foldings, the value $\lambda_2 = 1$ can then only be obtained if $\chi_0/f = \exp(-10^{11})$; the field must start ridiculously near the origin. The fraction of the universe which will start out with that value is tiny.

Thus, to obtain a reasonable probability of sufficient inflation, one must return to the values $f \sim m_{pl}$ and $\Lambda \sim 10^{-3} m_{pl}$ for the width and height of the rolling PNGB field. In that case, from eqn. (19) we have the parameter $\lambda_2 \sim 10^{-33}$, a very small number. Again, one sees the resurgence of the fine-tuning problem that pervades many inflationary models. Of course it may well be that there will someday be an explanation for this small number, e.g. it may be related to the solution to the heirarchy problem. However, here we have made it our task to consider a PNGB as the rolling field in order to try to avoid unexplained small numbers; yet, a small number surfaced in the potential for the other scalar field.

We also wish to check the validity of the gradient expansion. In eqn. (8) we have kept only the first order term and have neglected terms involving higher derivatives of $\dot\chi$. This is valid as long as $\dot\chi \ll f^2$. Using eqn. (15), we see that this roughly requires $\Lambda^2 \frac{m_{pl}}{f} \ll f^2$. For $m_{pl} \sim f$ this is definitely satisfied since $\Lambda/f \leq 10^{-3}$ is required [9].

In short, we have used a PNGB field as the rolling field in the Double Field Inflation model to naturally explain the required flatness of the potential for that field. However, the interaction term that we used in eqn. (16) leads to a new fine-tuning of the parameters of the potential for the inflaton field, which is a tunneling field. This new small number arises because the various constraints on the model require the heights of both potentials to be comparable and the widths of both potentials to be comparable. We suspect that this feature will be generic to many other attempts to implement the Double Field model, and that a small parameter may be difficult to avoid.

Other Effects: The term in eqns. (10) or (16) could serve to couple the QCD axion to other scalars, and one can check to see if there are any important effects. For instance, the other scalar could be Higgs or the bosons in non-topological soliton stars [11]; these scalars could in principle decay to axions (or to the PNGB in the case of inflation). However, the decay rates are extremely suppressed: $V_I \propto \frac{(\partial_\mu\chi)^2}{m_1^2}\Phi_2^2 \sim (\frac{m_S}{m_1})^2\Phi_2^2\chi^2$, where m_S is the scalar mass and m_1 in the denominator is the Peccei Quinn scale $\sim 10^{12}$ GeV for the case of the QCD axion (and $\sim 10^{19}$ GeV for the case of the inflaton). Because these couplings are so small we suspect they are unlikely to be important.

In summary, we found a derivative coupling of a PNGB to other scalars in eqn. (16). This coupling could facilitate Double Field Inflation with a PNGB as the rolling field that catalyzes a time dependent nucleation rate for the inflaton. The potential for the PNGB is then naturally flat enough to account for a small amplitude of density fluctuations. However, we found that the inflaton potential must also be flat. We also found that the same coupling term in the case of QCD axions is not likely to play a large role because it is suppressed.

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